# Geometric flows

David Glickenstein

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In this series of talks, we will consider some geometric evolution equations. An evolution equation is a differential equation (ODE/PDE) that takes some geometric object (curve, polygon, polyhedron, manifold, etc.) and changes it with time. We will primarily be considering geometric flows that have a smoothing effect: given an object with not much symmetry, we try to evolve it to one with more symmetry. We will consider several problems in this context:

1) Polygons. Given an ordered sequence of points in the plane or some $\mathbb{R}^n$, we can connect these points by line segments (and also the first to the last) to form a generalized polygon. In order to simplify this polygon, we evolve the points in some way in order to try to shrink it to a point in some uniform way. We will look at a very basic evolution of this type and see how its behavior can be characterized completely.

2) Curves. We will draw some analogies between the polygon flow and flows of smooth curves.

3) Surfaces and regions. We give a quick look at how to evolve surfaces into constant curvature surfaces, and the problem of evolving surfaces with boundary.

4) Triangulated surfaces. Given a manifold (possibly with boundary) obtained by gluing together Euclidean triangles, one can ask for the most symmetric version of this object, and define a flow of these objects in order to obtain such a symmetric object. One example is taking a triangulation of a region and evolving it to a triangulation of the disk (a triangulated version of the Riemann Mapping Theorem). Another example is taking a closed surface and evolving it to one of constant curvature.

6) Riemannian manifolds. We give a quick look at evolutions of manifolds. In particular, Ricci and Yamabe flows.

5) Triangulated 3-manifolds. Given a three-dimensional manifold obtained by gluing together Euclidean tetrahedra, is there a flow analogous to those considered on triangulated surfaces? We will introduce some open problems and some possible routes to solutions.